JEE–MAIN EXAMINATION – JANUARY 2025

(HELD ON THURSDAY 23rd JANUARY 2025)

TIME : 3:00 PM TO 6:00 PM

MATHEMATICS

SECTION-A

1. If in the expansion of $(1 + x)^p (1 - x)^q$, the coefficients of x and x^2 are 1 and –2, respectively, then $p^2 + q^2$ is equal to :

(1) 8 (2) 18

(3) 13 (4) 20

Ans. (3)

Sol. $(1+x)^p(1-x)^q = \left({}^pC_0 + {}^pC_1x + {}^pC_2x^2 + ...\right)\left({}^qC_0 - {}^qC_1x + {}^qC_2x^2 + ...\right)$

coff of $x \equiv {}^pC_0\,{}^qC_1 - {}^pC_1\,{}^qC_0 = 1$

$p - q = 1$

coff of $x^2 \equiv {}^pC_0\,{}^qC_2 - {}^pC_1\,{}^qC_1 + {}^pC_2\,{}^qC_0 = -2$

$$\frac{q(q-1)}{2} - pq + \frac{p(p-1)}{2} = -2$$

$q^2 - q - 2pq + p^2 - p = -4$

$(p - q)^2 - (p + q) = -4$

$p + q = 5$

$p = 3$

$q = 2$

so $p^2 + q^2 = 13$

2. Let A = {(x, y) ∈ **R** × **R** : |x + y| ≥ 3} and

B = {(x, y) ∈ **R** × **R** : |x| + |y| ≤ 3}.

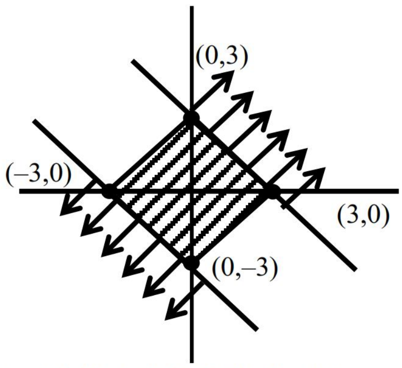
If C = {(x, y) ∈ **A** ∩ **B** : x = 0 or y = 0}, then $\sum_{(x,y)\in C} |x+y|$ is :

(1) 15 (2) 18

(3) 24 (4) 12

Ans. (4)

Sol.



C = {(3,0), (–3,0), (0,3), (0,–3)}

Σ|x + y| = 12

3. The system of equations

$x + y + z = 6,$

$x + 2y + 5z = 9,$

$x + 5y + \lambda z = \mu,$

has no solution if

(1) $\lambda = 17, \mu \neq 18$

(2) $\lambda \neq 17, \mu \neq 18$

(3) $\lambda = 15, \mu \neq 17$

(4) $\lambda = 17, \mu = 18$

Ans. (1)

Sol. $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 5 \\ 1 & 5 & \lambda \end{vmatrix} = 0$

$\lambda = 17$

$D_z = \begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 9 \\ 1 & 5 & \mu \end{vmatrix} \neq 0$

$\mu \neq 18$

4. Let $\int x^3 \sin x dx = g(x) + C$, where C is the constant of integration.

If $8\left(g\left(\frac{\pi}{2}\right) + g'\left(\frac{\pi}{2}\right)\right) = \alpha\pi^3 + \beta\pi^2 + \gamma$, $\alpha, \beta, \gamma \in Z$,

Then $\alpha + \beta - \gamma$ equals :

(1) 55 (2) 47

(3) 48 (4) 62

Ans. (1)

Sol. $\int x^3 \sin x dx = -x^3 \cos x + \int 3x^2 \cos x dx$

$= -x^3 \cos x + 3x^2 \sin x - \int 6x \sin x dx$

$= -x^3 \cos x + 3x^2 \sin x + 6x \cos x - 6 \sin x + c$

So $g(x) = -x^3 \cos x + 3x^2 \sin x + 6x \cos x - 6 \sin x$

$$g\left(\frac{\pi}{2}\right) = \frac{3\pi^2}{4} - 6$$

$g'(x) = x^3 \sin x$

$$g'\left(\frac{\pi}{2}\right) = \frac{\pi^3}{8}$$

$$8\left(g\left(\frac{\pi}{2}\right) + g'\left(\frac{\pi}{2}\right)\right) = \pi^3 + 6\pi^2 - 48$$

So $\alpha + \beta - \gamma = 55$

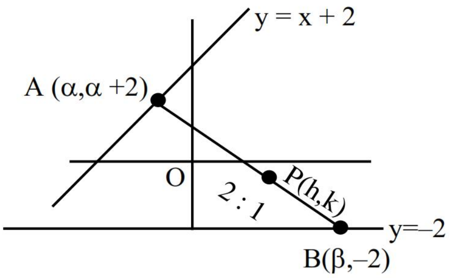
5. A rod of length eight units moves such that its ends A and B always lie on the lines $x - y + 2 = 0$ and $y + 2 = 0$, respectively. If the locus of the point P, that divides the rod AB internally in the ratio 2 : 1 is $9(x^2 + \alpha y^2 + \beta xy + \gamma x + 28y) - 76 = 0$, then $\alpha - \beta - \gamma$ is equal to :

(1) 24 (2) 23

(3) 21 (4) 22

Ans. **(2)**

Sol.



$$h = \frac{3\beta + \alpha}{3}$$

$$k = \frac{-4 + \alpha + 2}{3}$$

$\alpha = 3k + 2$

$2\beta = 3h - a = 3h - 3k - 2$

so AB = 8

$$(\alpha - \beta)^2 + (\alpha + 4)^2 = 64$$

$$\left(3k + 2 - \left(\frac{3h - 3k - 2}{2}\right)\right)^2 + (3k + 2 + 4)^2 = 64$$

$$\frac{(9k - 3h + 6)^2}{4} + (3k + 6)^2 = 64$$

$$9\left[(3k - h + 2)^2 + 4(k + 2)^2\right] = 64 \times 4$$

$9(x^2 + 13y^2 - 6xy - 4x + 28y) = 76$

$\alpha - \beta - \gamma = 13 + 6 + 4 = 23$

6. The distance of the line $\frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4}$ from the point (1, 4, 0) along the line $\frac{x}{1} = \frac{y-2}{2} = \frac{z+3}{3}$ is :

(1) $\sqrt{17}$ (2) $\sqrt{14}$

(3) $\sqrt{15}$ (4) $\sqrt{13}$

Ans. **(2)**

Sol. Let the parallel line is

$$\frac{x-1}{1} = \frac{y-4}{2} = \frac{z-0}{3}$$

so their point of intersection is

$(\lambda + 1, 2\lambda + 4\ 3\lambda) = (2t + 2, 3t + 6, 4t + 3)$

$\lambda = 2t + 1$

$2\lambda + 4 = 3t + 6 \Rightarrow t = 0$

so POI is (2,6,3)

so distance $= \sqrt{(2-1)^2 + (6-4)^2 + (3-0)^2} = \sqrt{14}$

7. Let the point A divide the line segment joining the points P(–1, –1, 2) and Q(5, 5,10) internally in the ratio r : 1 (r > 0). If O is the origin and $\left(\overrightarrow{OQ}.\overrightarrow{OA}\right) - \frac{1}{5}\left|\overrightarrow{OP} \times \overrightarrow{OA}\right|^2 = 10$, then the value of r is :

(1) 14 (2) 3

(3) $\sqrt{7}$ (4) 7

Ans. **(4)**

Sol. $A \equiv \left(\frac{5r-1}{r+1}, \frac{5r-1}{r+1}, \frac{10r+2}{r+1}\right)$

$$\left(\overrightarrow{OQ}.\overrightarrow{OA}\right) - \frac{\left|\overrightarrow{OP} \times \overrightarrow{OA}\right|^2}{5} = 10 \quad ....(1)$$

$$\overrightarrow{OQ}.\overrightarrow{OA} = \frac{10}{r+1}(15r + 1)$$

$$\left|\overrightarrow{OP} \times \overrightarrow{OA}\right|^2 = \frac{r^2}{(r+1)^2}(800)$$

so by equation (1)

$$\frac{10}{r+1}(15r + 1) - \frac{1}{5}\frac{r^2(800)}{(r+1)^2} = 10$$

$2r^2 - 14r = 0$

$r = 7, r \neq 0$

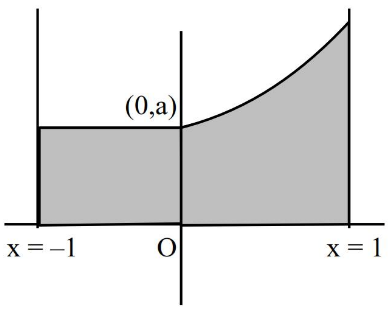
8. If the area of the region $\{(x, y) : -1 \le x \le 1, 0 \le y \le a + e^{|x|} - e^{-x}, a > 0\}$ is $\frac{e^2 + 8e + 1}{e}$, then the value of a is :

(1) 7 (2) 6

(3) 8 (4) 5

Ans. **(4)**

Sol.



required area is $a+\int_0^1\left(a+e^x-e^{-x}\right)dx$

$a+\left[a+e^x+e^{-x}\right]_0^1$

$2a+e-1+e^{-1}-1=e+8+\dfrac{1}{e}$

$2a = 10 \Rightarrow a = 5$

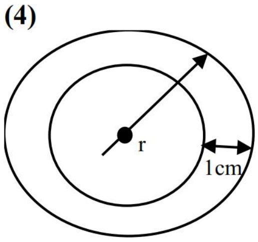
9. A spherical chocolate ball has a layer of ice-cream of uniform thickness around it. When the thickness of the ice-cream layer is 1 cm, the ice-cream melts at the rate of 81 cm^3/min and the thickness of the ice-cream layer decreases at the rate of $\dfrac{1}{4\pi}$ cm/min. The surface area (in cm^2) of the chocolate ball (without the ice-cream layer) is :

(1) 225π (2) 128π

(3) 196π (4) 256π

Ans. **(4)**

Sol



$v=\dfrac{4}{3}\pi r^3$

$\dfrac{dv}{dt}=4\pi r^2\dfrac{dr}{dt}$

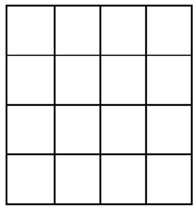
$81=4\pi r^2\times\dfrac{1}{4\pi}$

$r^2 = 81$

$r = 9$

surface area of chocolate $= 4\pi(r-1)^2 = 256\pi$

10. A board has 16 squares as shown in the figure :



Out of these 16 squares, two squares are chosen at random. The probability that they have no side in common is :

(1) $\dfrac{4}{5}$ (2) $\dfrac{7}{10}$

(3) $\dfrac{3}{5}$ (4) $\dfrac{23}{30}$

Ans. **(1)**

Sol. Total ways for selecting any two squares $= {}^{16}C_2$

$= 120$

Total ways for selecting common side squares

$=\underbrace{3\times4}_{\text{Horizontal side}}+\underbrace{3\times4}_{\text{vertical side}}$

$= 24$

so required probability

$=1-\dfrac{24}{120}$

$=\dfrac{4}{5}$

11. Let $x = x(y)$ be the solution of the differential equation

$y=\left(x-y\dfrac{dx}{dy}\right)\sin\left(\dfrac{x}{y}\right),\ y>0$ and $x(1)=\dfrac{\pi}{2}$.

Then $\cos(x(2))$ is equal to :

(1) $1-2(\log_e 2)^2$ (2) $2(\log_e 2)^2-1$

(3) $2(\log_e 2)-1$ (4) $1-2(\log_e 2)$

Ans. **(2)**

Sol. $ydy = (xdy - ydx)\sin\left(\dfrac{x}{y}\right)$

$\dfrac{dy}{y}=\left(\dfrac{xdy-ydx}{y^2}\right)\sin\left(\dfrac{x}{y}\right)$

$\dfrac{dy}{y}=\sin\left(\dfrac{x}{y}\right)d\left(-\dfrac{x}{y}\right)$

$\Rightarrow \quad \ell ny=\cos\dfrac{x}{y}+C$

$x(1) = \frac{\pi}{2} \Rightarrow 0 = \cos\frac{\pi}{2} + C \Rightarrow C=0$

$\ell ny = \cos\frac{x}{y}$

but $y = 2 \Rightarrow \cos\frac{x}{2} = \ell n2$

$\cos x = 2\cos^2\frac{x}{2} - 1$

$= 2(\ell n2)^2 - 1$

12. Let the range of the function

$f(x) = 6 + 16 \cos x \,.\, \cos\left(\frac{\pi}{3} - x\right) . \cos\left(\frac{\pi}{3} + x\right) .$

$\sin 3x \,.\, \cos 6x$, $x \in R$ be $[\alpha, \beta]$. Then the distance of the point (α, β) from the line $3x + 4y + 12 = 0$ is :

(1) 11 (2) 8

(3) 10 (4) 9

Ans. **(1)**

Sol. $f(x) = 6 + 16\left(\frac{1}{4}\cos 3x\right)\sin 3x.\cos 6x$

$= 6 + 4 \cos 3x \sin 3x \cos 6x$

$= 6 + \sin 12x$

Range of $f(x)$ is $[5, 7]$

$(\alpha, \beta) \equiv (5, 7)$

distance $= \left|\frac{15+28+12}{5}\right| = 11$

13. Let the shortest distance from $(a, 0)$, $a > 0$, to the parabola $y^2 = 4x$ be 4. Then the equation of the circle passing through the point $(a, 0)$ and the focus of the parabola, and having its centre on the axis of the parabola is:

(1) $x^2 + y^2 - 6x + 5 = 0$

(2) $x^2 + y^2 - 4x + 3 = 0$

(3) $x^2 + y^2 - 10x + 9 = 0$

(4) $x^2 + y^2 - 8x + 7 = 0$

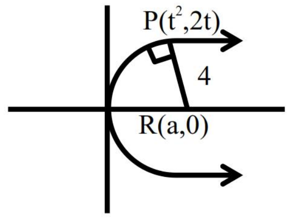
Ans. **(1)**

Sol. Normal at P

$y + tx = 2t + t^3$

$\uparrow$

$(a, 0)$



$at = 2t + t^3$

$a = 2 + t^2$

$R\ (2 + t^2, 0)$

$PR = 4 \Rightarrow 4 + 4t^2 = 16$

$4t^2 = 12 \Rightarrow t^2 = 3$

$a = 5$, $R(5, 0)$

Focus $(1, 0)$

$(1, 0)$ & $(5, 0)$ will be the end points of diameter

$\Rightarrow$ Eq^n of circle is

$(x - 1)(x - 5) + y^2 = 0$

$x^2 + y^2 - 6x + 5 = 0$

14. Let $X = R \times R$. Define a relation R on X as:

$(a_1, b_1)\ R\ (a_2, b_2) \Leftrightarrow b_1 = b_2$.

Statement-I: R is an equivalence relation.

Statement-II: For some $(a, b) \in X$, the set $S = \{(x, y) \in X : (x, y)\ R\ (a, b)\}$ represents a line parallel to $y = x$.

In the light of the above statements, choose the **correct** answer from the options given below:

(1) Both **Statement-I** and **Statement-II** are false.

(2) **Statement-I** is true but **Statement-II** is false.

(3) Both **Statement-I** and **Statement-II** are true.

(4) **Statement-I** is false but **Statement-II** is true.

Ans. **(2)**

Sol. **Statement – I :**

Reflexive : $(a_1,b_1)\ R(a_1, b_1) \Rightarrow b_1 = b_1$ True

Symmetric : $\left.\begin{array}{l}(a_1,b_1)\ R\ (a_2, b_2) \Rightarrow b_1 = b_2 \\ (a_2,b_2)\ R\ (a_1, b_1) \Rightarrow b_2 = b_1\end{array}\right\}$ True

Transitive : $\left.\begin{array}{l}(a_1, b_1)\ R\ (a_2, b_2) \Rightarrow b_1 = b_2 \\ \&\ (a_2, b_2)\ R\ (a_3, b_3)\ b_2 = b_3\end{array}\right\} b_1 = b_3$

$\Rightarrow (a_1, b_1)\ R\ (a_3, b_3) \Rightarrow$ True

Hence Relation R is an equivence relation Statement-I is true.

For statement – II $\Rightarrow y = b$ so False

15. The length of the chord of the ellipse $\frac{x^2}{4}+\frac{y^2}{2}=1$, whose mid-point is $\left(1,\frac{1}{2}\right)$, is:

(1) $\frac{2}{3}\sqrt{15}$ (2) $\frac{5}{3}\sqrt{15}$

(3) $\frac{1}{3}\sqrt{15}$ (4) $\sqrt{15}$

Ans. (1)

Sol. $T = S_1$

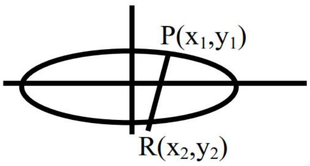
$$\frac{x.1}{4}+\frac{y.½}{2}=\frac{1}{4}+\frac{1}{8}$$

$$x+y=\frac{3}{2}$$

solve with ellipse

$$PR=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$=\sqrt{2}\left|x_2-x_1\right|$$



$$y_2=\frac{3}{2}-x_2$$

$$y_1=\frac{3}{2}-x_1$$

$$y_2-y_1=x_2-x_1$$

$$x^2+2y^2=4$$

$$x^2+2\left(\frac{3}{2}-x\right)^2=4$$

$$6x^2-12x+1=0$$

$$x_1+x_2=2$$

$$x_1x_2=1/6$$

$$\left|x_2-x_1\right|=\sqrt{(x_2+x_1)^2-4x_1x_2}$$

$$=\sqrt{4-4/6}$$

$$PR=\sqrt{2}.2.\frac{\sqrt{5}}{\sqrt{2}\sqrt{3}}=\frac{2}{3}\sqrt{15}$$

16. Let $A=[a_{ij}]$ be a 3×3 matrix such that $A\begin{bmatrix}0\\1\\0\end{bmatrix}=\begin{bmatrix}0\\0\\1\end{bmatrix}$, $A\begin{bmatrix}4\\1\\3\end{bmatrix}=\begin{bmatrix}0\\1\\0\end{bmatrix}$ and $A\begin{bmatrix}2\\1\\2\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$, then a_{23} equals:

(1) –1 (2) 0

(3) 2 (4) 1

Ans. (1)

Sol. Let $A=\begin{bmatrix}a_{11}&a_{12}&a_{13}\\a_{21}&a_{22}&a_{23}\\a_{31}&a_{32}&a_{33}\end{bmatrix}$

$$A\begin{bmatrix}0\\1\\0\end{bmatrix}=\begin{bmatrix}0\\0\\1\end{bmatrix}\Rightarrow\begin{bmatrix}a_{12}\\a_{22}\\a_{32}\end{bmatrix}=\begin{bmatrix}0\\1\\0\end{bmatrix}\Rightarrow \begin{matrix}a_{22}=0;a_{12}=0\\a_{32}=1\end{matrix}$$

$$A\begin{bmatrix}4\\1\\3\end{bmatrix}=\begin{bmatrix}0\\1\\0\end{bmatrix}\Rightarrow\begin{matrix}4a_{11}+a_{12}+3a_{13}=0\\4a_{21}+a_{22}+3a_{23}=1\Rightarrow4a_{21}+3a_{23}=1\\4a_{31}+a_{32}+3a_{33}=0\end{matrix}$$

$$A\begin{bmatrix}2\\1\\2\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}\Rightarrow\begin{matrix}2a_{11}+a_{12}+2a_{13}=1\\2a_{21}+a_{22}+2a_{23}=0\Rightarrow a_{21}+a_{23}=0\\2a_{31}+a_{32}+2a_{33}=0\end{matrix}$$

$$-4a_{23}+3a_{23}=1\Rightarrow a_{23}=-1$$

17. The number of complex numbers z, satisfying $|z| = 1$ and $\left|\frac{z}{\bar{z}}+\frac{\bar{z}}{z}\right|=1$, is :

(1) 6 (2) 4

(3) 10 (4) 8

Ans. (4)

Sol. $z=e^{i\theta}$

$$\frac{z}{\bar{z}}=e^{i2\theta}$$

$$\left|\frac{z}{\bar{z}}+\frac{\bar{z}}{z}\right|=1\Rightarrow\left|e^{i2\theta}+e^{-12\theta}\right|=1\Rightarrow\left|\cos2\theta\right|=\frac{1}{2}$$

8 solution in $[0, 2\pi)$

18. If the square of the shortest distance between the lines $\frac{x-2}{1}=\frac{y-1}{2}=\frac{z+3}{-3}$ and $\frac{x+1}{2}=\frac{y+3}{4}=\frac{z+5}{-5}$ is $\frac{m}{n}$, where m, n are coprime numbers, then m + n is equal to:

(1) 6 (2) 9

(3) 21 (4) 14

Ans. (2)

Sol. $\vec{a}=(2,1,-3)$

$\vec{b}=(-1,-3,-5)$

$$\vec{p}\times\vec{q}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & -3\\ 2 & 4 & -5\end{vmatrix}$$

$=2\hat{i}-\hat{j}$

$\vec{b}-\vec{a}=-3\hat{i}-4\hat{j}-2\hat{k}$

$$S_d=\frac{\left|(\vec{b}-\vec{a}).(\vec{p}\times\vec{q})\right|}{|\vec{p}\times\vec{q}|}$$

$=\frac{2}{\sqrt{5}}$

$(S_d)^2=\frac{4}{5}$

$m=4,\ n=5\Rightarrow m+n=9$

19. If $I=\int_0^{\frac{\pi}{2}}\frac{\sin^{\frac{3}{2}}x}{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}dx$,

then $\int_0^{21}\frac{x\sin x\cos x}{\sin^4 x+\cos^4 x}dx$ equals:

(1) $\frac{\pi^2}{16}$ (2) $\frac{\pi^2}{4}$

(3) $\frac{\pi^2}{8}$ (4) $\frac{\pi^2}{12}$

Ans. (1)

Sol. For I

Apply king (P– 5) and add

$2I=\int_0^{\pi/2}dx=\frac{\pi}{2}\Rightarrow I=\frac{\pi}{4}$

$I_2=\int_0^{\pi/2}\frac{x\sin x\cos x}{\sin^4 x+\cos^4 x}dx$

Apply king and add

$I_2=\frac{\pi}{4}\int_0^{\pi/2}\frac{\tan x\sec^2 x dx}{\tan^4 x+1}$

put $\tan^2 x=t$

$\frac{\pi}{8}\int_0^{\infty}\frac{dt}{t^2+1}$

$=\frac{\pi}{8}.\frac{\pi}{2}=\frac{\pi^2}{16}$

20. $\lim_{x\to\infty}\frac{(2x^2-3x+5)(3x-1)^{\frac{x}{2}}}{(3x^2+5x+4)\sqrt{(3x+2)^x}}$ is equal to:

(1) $\frac{2}{\sqrt{3e}}$ (2) $\frac{2e}{\sqrt{3}}$

(3) $\frac{2e}{3}$ (4) $\frac{2}{3\sqrt{e}}$

Ans. (4)

Sol. $\lim_{x\to\infty}\frac{\left(2-\frac{3}{x}+\frac{5}{x^2}\right)\left(1-\frac{1}{3x}\right)^{x/2}}{\left(3+\frac{5}{x}+\frac{4}{x^2}\right)\left(1+\frac{2}{3x}\right)^{x/2}}$

$=\lim_{x\to\infty}\frac{2}{3}.\frac{e^{\frac{x}{2}\left(1-\frac{1}{3x}-1\right)}}{e^{\frac{x}{2}\left(1+\frac{2}{3x}-1\right)}}$

$=\frac{2}{3}.\frac{e^{-\frac{1}{6}}}{e^{1/3}}=\frac{2}{3}e^{-\frac{1}{2}}$

SECTION-B

21. The number of ways, 5 boys and 4 girls can sit in a row so that either all the boys sit together or no two boys sit together, is ________.

Ans. (17280)

Sol. A : number of ways that all boys sit together = 5! × 5!

B : number of ways if no 2 boys sit together = 4! × 5!

$A\cap B=\phi$

Required no. of ways = 5! × 5! + 4! × 5! = 17280

22. Let α, β be the roots of the equation $x^2-ax-b=0$ with $Im(\alpha)<Im(\beta)$. Let $P_n=\alpha^n-\beta^n$. If $P_3=-5\sqrt{7}i$, $P_4=-3\sqrt{7}i$, $P_5=11\sqrt{7}i$ and $P_6=45\sqrt{7}i$, then $|\alpha^4+\beta^4|$ is equal to ________.

Ans. (31)

Sol. $\alpha+\beta=a$ $\alpha\beta=-b$

$P_6=aP_5+bP_4$

$45\sqrt{7}i=a\times11\sqrt{7}i+b(-3\sqrt{7})i$

$45=11a-3b$ ...(1)

and

$P_5 = aP_4 + bP_3$

$$11\sqrt{7}i = a\left(-3\sqrt{7}i\right) + b\left(-5\sqrt{7}i\right)$$

$11 = -3a - 5b$ ...(2)

$a = 3, b = -4$

$$\left|\alpha^4 + \beta^4\right| = \sqrt{\left(\alpha^4 - \beta^4\right)^2 + 4\alpha^4\beta^4}$$

$$= \sqrt{-63 + 4.4^4}$$

$$= \sqrt{-63 + 1024} = \sqrt{961} = 31$$

23. The focus of the parabola $y^2 = 4x + 16$ is the centre of the circle C of radius 5. If the values of λ, for which C passes through the point of intersection of the lines $3x - y = 0$ and $x + \lambda y = 4$, are λ_1 and λ_2, $\lambda_1 < \lambda_2$, then $12\lambda_1 + 29\lambda_2$ is equal to _______.

Ans. **(15)**

Sol. $y^2 = 4(x + 4)$

Equation of circle

$(x + 3)^2 + y^2 = 25$

Passes through the point of intersection of two lines $3x - y = 0$ and $x + \lambda y = 4$ which is

$\left(\frac{4}{3\lambda+1}, \frac{12}{3\lambda+1}\right)$, after solving with circle,

we get

$$\lambda = -\frac{7}{6}, 1$$

$$12\lambda_1 + 29\lambda_2$$

$-14 + 29 = 15$

24. The variance of the numbers 8, 21, 34, 47, …, 320, is ________.

Ans. **(8788)**

Sol. Var(8, 21, 34, 47, ..., 320)

Var(0, 13, 26, 39, ..., 312)

13^2.Var(0, 1, 2, ..., 24)

13^2.Var(1, 2, 3, ..., 25)

So, $\sigma^2 = 13^2 \times \left(\frac{25^2 - 1}{12}\right) = 8788$

Alternate solution

$8 + (n - 1)13 = 320$

$13n = 325$

$n = 25$

no. of terms = 25

$$\text{mean} = \frac{\Sigma x_i}{n} = \frac{8 + 21 + ... + 320}{25} = \frac{\frac{25}{2}(8 + 320)}{25}$$

$$\text{variance } \sigma^2 = \frac{\Sigma x_i^2}{n} - (\text{mean})^2$$

$$= \frac{8^2 + 21^2 + ... + 320^2}{13} - (164)^2$$

$= 8788$

25. The roots of the quadratic equation $3x^2 - px + q = 0$ are 10^{th} and 11^{th} terms of an arithmetic progression with common difference $\frac{3}{2}$. If the sum of the first 11 terms of this arithmetic progression is 88, then $q - 2q$ is equal to __________.

Ans. **(474)**

Sol. $S_{11} = \frac{11}{2}(2a + 10d) = 88$

$a + 5d = 8$

$$a = 8 - 5 \times \frac{3}{2} = \frac{1}{2}$$

Roots are

$$T_{10} = a + 9d = \frac{1}{2} + 9 \times \frac{3}{2} = 14$$

$$T_{11} = a + 10d = \frac{1}{2} + 10 \times \frac{3}{2} = \frac{31}{2}$$

$$\frac{p}{3} = T_{10} + T_{11} = 14 + \frac{31}{2} = \frac{59}{2}$$

$$p = \frac{177}{2}$$

$$\frac{q}{3} = T_{10} \times T_{11} = 7 \times 31 = 217$$

$q = 651$

$q - 2p$

$= 651 - 177$

$= 474$

PHYSICS

SECTION-A

26. A ball having kinetic energy KE, is projected at an angle of 60° from the horizontal. What will be the kinetic energy of ball at the highest point of its flight?

(1) $\frac{(KE)}{8}$ (2) $\frac{(KE)}{4}$

(3) $\frac{(KE)}{16}$ (4) $\frac{(KE)}{2}$

Ans. (2)

Sol. Initial K.E,

$$K.E. = \frac{1}{2}mu^2$$

Speed at heighest point

$$V = u\cos 60° = \frac{u}{2}$$

$$\therefore\ KE_2 = \frac{1}{2}m\left(\frac{u}{2}\right)^2$$

$$= \frac{1}{4}\times\frac{1}{2}mu^2$$

$$= \frac{KE}{4}$$

27. Two charges 7 μc and – 4 μc are placed at (– 7 cm, 0, 0) and (7 cm, 0, 0) respectively. Given, $\epsilon_0 = 8.85 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$, the electrostatic potential energy of the charge configuration is :

(1) – 1.5 J (2) – 2.0 J

(3) – 1.2 J (4) – 1.8 J

Ans. (4)

Sol. P.E. of two charges

$$u = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$$

$$r = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

$= 14$ cm

$$\therefore u = \frac{9\times10^9\times7\times10^{-6}\times(-4)\times10^{-6}}{14\times10^{-2}}$$

$= -1.8$ J

28. The refractive index of the material of a glass prism is $\sqrt{3}$. The angle of minimum deviation is equal to the angle of the prism. What is the angle of the prism?

(1) 50° (2) 60°

(3) 58° (4) 48°

Ans. (2)

Sol. $$\mu = \frac{\sin\left(\frac{A+\delta_{min}}{2}\right)}{\sin\frac{A}{2}}$$

Given $\delta_{min} = A$

$$\sqrt{3} = \frac{\sin A}{\sin\frac{A}{2}} = \frac{2\sin\frac{A}{2}\cos\frac{A}{2}}{\sin\frac{A}{2}}$$

$$\cos\frac{A}{2} = \frac{\sqrt{3}}{2}$$

$A = 60°$

29. The equation of a transverse wave travelling along a string is $y(x, t) = 4.0\sin[20\times10^{-3}x + 600t]$ mm, where x is in the mm and t is in second. The velocity of the wave is :

(1) + 30 m/s (2) – 60 m/s

(3) – 30 m/s (4) + 60 m/s

Ans. (3)

Sol. $y = 4\sin(20\times10^{-3}x + 600t)$

Here $\omega = 600\ s^{-1}$

$k = 20\times10^{-3}\ mm^{-1}$

$$\therefore\ v = \frac{w}{k} = \frac{600}{20\times10^{-3}}$$

$= 30\times10^3$ mm/s

$= 30$ m/s

& direction is towards -ve x axis

$\therefore\ v = -30$ m/s

30. The energy of a system is given as $E(t) = \alpha^3 e^{-\beta t}$, where t is the time and $\beta = 0.3\ s^{-1}$. The errors in the measurement of α and t are 1.2% and 1.6%, respectively. At t = 5 s, maximum percentage error in the energy is :

(1) 4% (2) 11.6%

(3) 6% (4) 8.4%

Ans. **(3)**

Sol. $E = \alpha^3 e^{-\beta t}$

$\ln E = 3 \ln \alpha - \beta t$

$$\left(\frac{dE}{E}\right)_{max} = \frac{3d\alpha}{\alpha} + \beta \frac{dt}{t} \times t$$

$= 3 \times 1.2\% + (0.3 \times 1.6 \times 5)\%$

$= 6\%$

31. In photoelectric effect an em-wave is incident on a metal surface and electrons are ejected from the surface. If the work function of the metal is 2.14 eV and stopping potential is 2V, what is the wavelength of the em-wave?

(Given hc = 1242 eVnm where h is the Planck's constant and c is the speed of light in vaccum.)

(1) 400 nm (2) 600 nm

(3) 200 nm (4) 300 nm

Ans. **(4)**

Sol. $eV_s = E - \phi$

$2\ eV = E - 2.14\ eV$

$E = 4.14\ eV$

$$E = \frac{hc}{\lambda}$$

$$\lambda = \frac{1242}{4.14} = 300\ nm$$

32. A circular disk of radius R meter and mass M kg is rotating around the axis perpendicular to the disk. An external torque is applied to the disk such that $\theta(t) = 5t^2 - 8t$, where $\theta(t)$ is the angular position of the rotating disc as a function of time t.

How much power is delivered by the applied torque, when t = 2s?

(1) $60\ MR^2$ (2) $72\ MR^2$

(3) $108\ MR^2$ (4) $8\ MR^2$

Ans. **(1)**

Sol. $\theta = 5t^2 - 8t$

$$\omega = \frac{d\theta}{dt} = 10\,t - 8$$

$$\alpha = \frac{d\omega}{dt} = 10$$

$\therefore\ p = \tau\omega$

$= (I\alpha)\,\omega$

$$= \left(\frac{mR^2}{2}\right)\alpha\,\omega$$

$$= \left(\frac{mR^2}{2}\right)(10)\,(10t - 8)$$

Put t = 2

$p = 60\ mR^2$

33. Water flows in a horizontal pipe whose one end is closed with a valve. The reading of the pressure gauge attached to the pipe is P_1. The reading of the pressure gauge falls to P_2 when the valve is opened. The speed of water flowing in the pipe is proportional to

(1) $\sqrt{P_1 - P_2}$ (2) $(P_1 - P_2)^2$

(3) $(P_1 - P_2)^4$ (4) $P_1 - P_2$

Ans. **(1)**

Sol. By Bernoulli equation

$$P_1 + \frac{1}{2} \times \rho \times 0^2 = P_2 + \frac{1}{2}\rho V^2$$

$$v = \sqrt{2\rho(P_1 - P_2)}$$

34. Match **List-I** with **List-II**.

List-I	List-II
(A) Permeability of free space	(I) $[M\,L^2T^{-2}]$
(B) Magnetic field	(II) $[M\,T^{-2}A^{-1}]$
(C) Magnetic moment	(III) $[M\,L\,T^{-2}\,A^{-2}]$
(D) Torsional constant	(IV) $[L^2\,A]$

Choose the **correct** answer from the options given below :

(1) (A)-(I), (B)-(IV), (C)-(II), (D)-(III)

(2) (A)-(II), (B)-(I), (C)-(III), (D)-(IV)

(3) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

(4) (A)-(III), (B)-(II), (C)-(IV), (D)-(I)

Ans. **(4)**

Sol. $B = \dfrac{\mu_0 I}{2\pi r}$

$\Rightarrow [\mu_0] = \left[\dfrac{B \times r}{I}\right] = \left[\dfrac{MT^{-2}A^{-1} \times L}{A}\right] = [MLT^{-2}A^{-2}]$

magnetic field $F = qvB$

$B = \left[\dfrac{MLT^{-2}}{AT\ L/T}\right] = [MT^{-2}A^{-1}]$

$[M] = [NTA] = [M] = [ML^2]$

$\tau = c\theta \Rightarrow c = \left[\dfrac{\tau}{\theta}\right] = [ML^2T^{-2}]$

35. If a satellite orbiting the Earth is 9 times closer to the Earth than the Moon, what is the time period of rotation of the satellite? Given rotational time period of Moon = 27 days and gravitational attraction between the satellite and the moon is neglected.

(1) 1 day (2) 81 days

(3) 27 days (4) 3 days

Ans. **(1)**

Sol. $T^2 \propto R^3$

$\left(\dfrac{T_m}{T_s}\right)^2 = \left(\dfrac{R}{R/9}\right)^3$

$\dfrac{T_m}{T_s} = (3)^3$

$\Rightarrow T_s = \left(\dfrac{27}{27}\right) = 1$ day

36. Two point charges – 4 μc and 4 μc, constituting an electric dipole, are placed at (– 9, 0, 0) cm and (9, 0, 0) cm in a uniform electric field of strength 10^4 NC^{-1}. The work done on the dipole in rotating it from the equilibrium through 180° is :

(1) 14.4 mJ (2) 18.4 mJ

(3) 12.4 mJ (4) 16.4 mJ

Ans. **(1)**

Sol. $U = -PE\cos\theta$

$w_{ext} = \Delta U = U_f - U_i = -PE\cos 180° + PE\cos 0°$

$w_{ext} = 2PE$

$= 2 \times (4 \times 10^{-6})(18) \times 10^4$

$= 144 \times 10^{-2}$

$= 14.4$ mJ

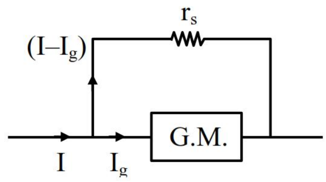
37. A galvanometer having a coil of resistance 30 Ω need 20 mA of current for full-scale deflection. If a maximum current of 3 A is to be measured using this galvanometer, the resistance of the shunt to be added to the galvanometer should be $\dfrac{30}{X}\Omega$, where X is

(1) 447 (2) 298

(3) 149 (4) 596

Ans. **(3)**

Sol.



$I_g R_g = (I - I_g) r_s$

$20 \times 10^{-3} \times 30 = (3 - 0.02) \times r_s$

$r_s = \left(\dfrac{0.6}{2.98}\right) = \dfrac{30}{x}$

$x = \left(\dfrac{2.98 \times 30}{0.6}\right) = 149$

38. The width of one of the two slits in Young's double slit experiment is d while that of the other slit is xd. If the ratio of the maximum to the minimum intensity in the interference pattern on the screen is 9 : 4 then what is the value of x?

(Assume that the field strength varies according to the slit width.)

(1) 2 (2) 3

(3) 5 (4) 4

Ans. **(3)**

Sol. $I \propto (\text{width})^2$

$\left(\dfrac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2 = \dfrac{9}{4}$

$\dfrac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}} = \dfrac{3}{2}$

$\dfrac{(x+1)d}{(x-1)d} = \dfrac{3}{2}$

$\Rightarrow 3x - 3 = 2x + 2$

$x = 5$

39. Given below are two statements. One is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : The binding energy per nucleon is found to be practically independent of the atomic number A, for nuclei with mass numbers between 30 and 170.

Reason (R) : Nuclear force is long range.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) **(A)** is false but **(R)** is true

(2) **(A)** is true but **(R)** is false

(3) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**

(4) Both **(A)** and **(R)** are true but **(R)** is **NOT** the correct explanation of **(A)**

Ans. **(2)**

Sol. Conceptual

40. Water of mass m gram is slowly heated to increase the temperature from T_1 to T_2. The change in entropy of the water, given specific heat of water is 1 $Jkg^{-1}K^{-1}$, is :

(1) zero (2) $m(T_2-T_1)$

(3) $m\,ln\left(\frac{T_1}{T_2}\right)$ (4) $m\,ln\left(\frac{T_2}{T_1}\right)$

Ans. **(4)**

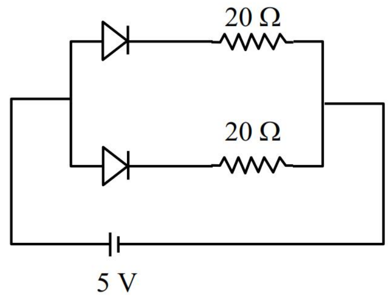
Sol. $dQ = msdT$

$$dS = \frac{dQ}{T} = \frac{msdT}{T}$$

$$\Delta S = \int \frac{msdT}{T} = ms\,ln\frac{T_f}{T_i}$$

$$\Delta S = m\,ln\frac{T_2}{T_1}$$

41. What is the current through the battery in the circuit shown below?



(1) 1.0 A (2) 1.5 A

(3) 0.5 A (4) 0.25 A

Ans. **(3)**

Sol. Both are forward biased

hence $R_{eq} = 10\ \Omega$

$$i = \frac{V}{R} = \frac{5}{10} = \frac{1}{2}A$$

42. A plane electromagnetic wave of frequency 20 MHz travels in free space along the +x direction. At a particular point in space and time, the electric field vector of the wave is $E_y = 9.3\ Vm^{-1}$. Then, the magnetic field vector of the wave at that point is-

(1) $B_z = 9.3 \times 10^{-8}$ T (2) $B_z = 1.55 \times 10^{-8}$ T

(3) $B_z = 6.2 \times 10^{-8}$ T (4) $B_z = 3.1 \times 10^{-8}$ T

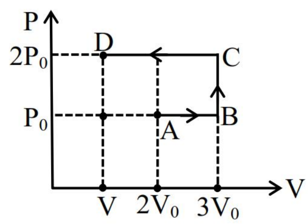
Ans. **(4)**

Sol. $E = BC$

$9.3 = B \times 3 \times 10^8$

$$B = \frac{9.3}{3\times10^8} = 3.1 \times 10^{-8}\ T$$

43.



Using the given P-V diagram, the work done by an ideal gas along the path ABCD is-

(1) $4\,P_0V_0$ (2) $3\,P_0V_0$

(3) $-4\,P_0V_0$ (4) $-3\,P_0V_0$

Ans. **(4)**

Sol. $w_{ABCD} = w_{AB} + w_{BC} + w_{CD}$

$= P_0V_0 + 0 + (-2P_0 \times 2V_0)$

$= P_0V_0 - 4P_0V_0$

$= -3P_0V_0$

44. A concave mirror of focal length f in air is dipped in a liquid of refractive index μ. Its focal length in the liquid will be :

(1) $\frac{f}{\mu}$ (2) $\frac{f}{(\mu-1)}$

(3) μf (4) f

Ans. **(4)**

Sol. Focal length of mirror will not change because focal length of mirror doesn't depend on medium.

45. A massless spring gets elongated by amount x_1 under a tension of 5N. Its elongation is x_2 under the tension of 7N. For the elongation of $(5x_1 - 2x_2)$, the tension in the spring will be,

(1) 15 N (2) 20 N

(3) 11 N (4) 39 N

Ans. **(3)**

Sol. $kx_1 = 5N$

$kx_2 = 7N$

$k(5x_1 - 2x_2) = 5kx_1 - 2kx_2$

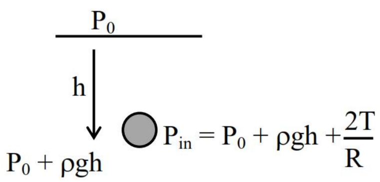
$= 5 \times 5 - 2 \times 7 = 11$ N

SECTION-B

46. An air bubble of radius 1.0 mm is observed at a depth of 20 cm below the free surface of a liquid having surface tension 0.095 J/m^2 and density 10^3 kg/m^3. The difference between pressure inside the bubble and atmospheric pressure_______ N/m^2. (Take g = 10 m/s^2)

Ans. **(2190)**

Sol.



$\Delta P = P_{in} - P_0$

$= \rho gh + \frac{2T}{R} = \frac{1000 \times 10 \times 20}{100} + \frac{2 \times 0.095}{10^{-3}}$

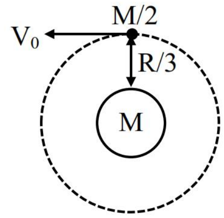
$= 2000 + 190$

$= 2190$

47. A satellite of mass $\frac{M}{2}$ is revolving around earth in a circular orbit at a height of $\frac{R}{3}$ from earth surface. The angular momentum of the satellite is $M\sqrt{\frac{GMR}{x}}$. The value of x is _______, where M and R are the mass and radius of earth, respectively. (G is the gravitational constant)

Ans. **(3)**

Sol. **(i)** If earth is assumed to be stationary



orbital velocity $v_0 = \sqrt{\frac{GM}{4R/3}} = \sqrt{\frac{3GM}{4R}}$

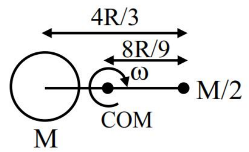
Angular momentum of satellite $= \frac{M}{2} v_0 \frac{4R}{3}$

$= \frac{M}{2} \cdot \sqrt{\frac{3GM}{4R}} \cdot \frac{4R}{3}$

$= M\sqrt{\frac{GMR}{3}}$

x = 3

(ii) Since mass of satellite is comparable to the mass of earth.



$$\frac{G.M.\frac{M}{2}}{\left(\frac{4R}{3}\right)^2} = \frac{M}{2}\omega^2 \cdot \frac{8R}{9}$$

$$\omega = \sqrt{\frac{81GM}{128R^3}}$$

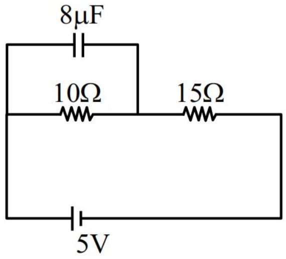
Angular momentum of satellite about common centre of mass,

$$L = \frac{M}{2} \cdot \left(\frac{8R}{9}\right)^2 \cdot \omega$$

$$L = M\sqrt{GMR\left(\frac{8}{81}\right)}$$

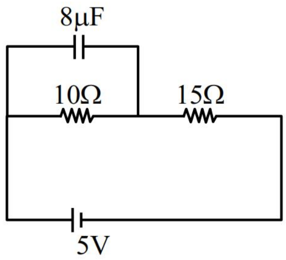
$$x = \frac{81}{8} \simeq 10$$

48. At steady state the charge on the capacitor, as shown in the circuit below, is _______ μC.



Ans. **(16)**

Sol.



$$i = \left(\frac{5}{25}\right)$$

$$Q = CV$$

$$Q = (8 \times 10^{-6})\left(\frac{5}{25} \times 10\right)$$

$$Q = \left(\frac{8 \times 5 \times 10^{-2}}{25}\right) = 16\ \mu C$$

49. A time varying potential difference is applied between the plates of a parallel plate capacitor of capacitance 2.5 μF. The dielectric constant of the medium between the capacitor plates is 1. It produces an instantaneous displacement current of 0.25 mA in the intervening space between the capacitor plates, the magnitude of the rate of change of the potential difference will be ________ Vs^{-1}.

Ans. **(100)**

Sol. $$\frac{CdV}{dt} = I_d$$

$$\frac{dV}{dt} = \frac{I_d}{C}$$

$$= \frac{0.25 \times 10^{-3}}{2.5 \times 10^{-6}}$$

$$= 100$$

50. In a series LCR circuit, a resistor of 300 Ω, a capacitor of 25 nF and an inductor of 100 mH are used. For maximum current in the circuit, the angular frequency of the ac source is _____ $\times 10^4$ radians s^{-1}.

Ans. **(2)**

Sol. $$\omega = \frac{1}{\sqrt{LC}}$$

$$\omega = \frac{1}{\sqrt{25 \times 10^{-9} \times 100 \times 10^{-3}}}$$

$$\omega = \frac{10^{+6}}{5 \times 10} = 2$$

CHEMISTRY

SECTION-A

51. The effect of temperature on spontaneity of reactions are represented as:

	ΔH	ΔS	Temperature	Spontaneity
(A)	+	–	any T	Non spontaneous
(B)	+	+	low T	spontaneous
(C)	–	–	low T	Non spontaneous
(D)	–	+	any T	spontaneous

(1) (B) and (D) only
(2) (A) and (D) only
(3) (B) and (C) only
(D) (A) and (C) only

Ans. **(3)**

Sol. $\because \Delta G = \Delta H - T\Delta S$

For spontaneity of reaction : $\Delta G = -ve$

52. Standard electrode potentials for a few half cells are mentioned below:

$E^{o}_{Cu^{2+}/Cu} = 0.34V, E^{o}_{Zn^{2+}/Zn} = -0.76V$

$E^{o}_{Ag^{+}/Ag} = 0.80V, E^{o}_{Mg^{2+}/Mg} = -2.37V$

Which one of the following cells gives the most negative value of ΔG^{o} ?

(1) $Zn|Zn^{2+}$ (1M)||Ag^{+}(1M) | Ag
(2) $Zn|Zn^{2+}$ (1M)||Mg^{2+}(1M) | Mg
(3) $Ag|Ag^{+}$ (1M)||Mg^{2+}(1M) | Mg
(4) Cu | Cu^{2+} (1M)||Ag^{+}(1M) | Ag

Ans. **(1)**

Sol. $\because \Delta G° = -nFE°$

Option (1) $E° = 0.8 + 0.76$
$= 1.56$ V

$\therefore \Delta G° = -2 \times F \times 1.56$
$= -3.12$ V

Option (2) $E° = -2.37 + 0.76$
$= -1.61$ V

$\therefore \Delta G° = -2 \times F \times (-1.61)$
$= +3.22$ V

Option (3) $E° = -2.37 - 0.8$
$= -3.17$ V

$\therefore \Delta G° = -2 \times F \times (-3.17)$
$= +6.34$

Option (4) $E° = 0.8 - 0.34$
$= 0.46$ V

$\Delta G° = -2 \times F \times 0.46$
$= -0.92$ V

53. The α - Helix and β - Pleated sheet structures of protein are associated with its:

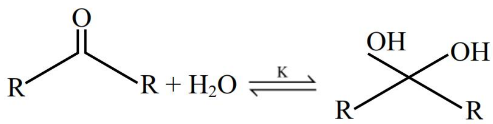
(1) quaternary structure
(2) primary structure
(3) secondary structure
(4) tertiary structure

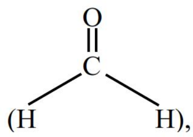
Ans. **(3)**

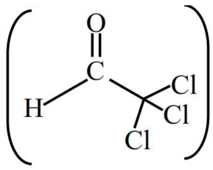
Sol. α-helix and β-pleated sheet belongs to secondary structure of protein, which have hydrogen bonds.

54. Given below are two statements:

Consider the following reaction



Statement (I) : In the case of formaldehyde , K is about 2280, due to small substituents, hydration is faster.

Statement (II) : In the case of trichloro acetaldehyde , K is about 2000 due to – I effect of – Cl.

In the light of the above statements, choose the **correct** answer from the options given below:

(1) **Statement I** true but **Statement II** is false
(2) Both **Statement I** and **Statement II** are true
(3) **Statement I** is false but **Statement II** is true
(4) Both **Statement I** and **Statement II** are false

Ans. **(2)**

Sol. $k_{eq} = 2280$ is for HCHO

$k_{eq} = 2000$ is for chloral

Both data is given in clayden and warren book.

$k_{eq} > 1$ because HCHO and chloral are more electrophilic.

55. Consider the reaction

$$X_2Y(g) \rightleftharpoons X_2(g) + \frac{1}{2}Y_2(g)$$

The equation representing correct relationship between the degree of dissociation (x) of $X_2Y(g)$ with its equilibrium constant Kp is________.
Assume x to be very very small.

(1) $x = \sqrt[3]{\frac{2Kp}{p}}$ (2) $x = \sqrt[3]{\frac{2Kp^2}{p}}$

(3) $x = \sqrt[3]{\frac{Kp}{2p}}$ (4) $x = \sqrt[3]{\frac{Kp}{p}}$

Ans. (2)

Sol. 1 mole

$$X_2Y_{(g)} \rightleftharpoons X_{2(g)} + \frac{1}{2}Y_{2(g)}$$

1-x mole x mole $\frac{x}{2}$ mole

$$\therefore\ P_{X_2Y} = \frac{1-x}{1+\frac{x}{2}} \times P$$

$$P_{X_2} = \frac{x}{1+\frac{x}{2}} \times P$$

$$P_{Y_2} = \frac{x/2}{1+\frac{x}{2}} \times P$$

$$\therefore\ K_p = \left(\frac{x}{1+\frac{x}{2}}P\right)\left(\frac{x}{2\left(1+\frac{x}{2}\right)}P\right)^{\frac{1}{2}} \Bigg/ \left(\frac{1-x}{1+\frac{x}{2}}\right) \times P$$

$$\therefore\ K_p = \left(\frac{x}{1-x}\right)\left(\frac{x}{2\left(1+\frac{x}{2}\right)}\right)^{\frac{1}{2}} \times p^{\frac{1}{2}}$$

$\because$ x to be very very small

$$\therefore\ K_p = \frac{x^{3/2}}{(2)^{\frac{1}{2}}} \times P^{\frac{1}{2}}$$

$$\therefore\ x^{\frac{3}{2}} = \frac{K_p \times 2^{\frac{1}{2}}}{P^{\frac{1}{2}}}$$

$$\therefore\ x^3 = \frac{K_p^2 \times 2}{P}$$

$$x = \left(\frac{K_p^2 \times 2}{P}\right)^{\frac{1}{3}}$$

56. Identify A, B and C in the given below reaction sequence

$$\boxed{A} \xrightarrow{HNO_3} Pb(NO_3)_2 \xrightarrow{H_2SO_4} \boxed{B} \xrightarrow[\text{(3) } K_2CrO_4]{\text{(1) Ammonium acetate (2) Acetic acid}} \boxed{C}$$

Yellow ppt

(1) $PbCl_2$, $PbSO_4$, $PbCrO_4$

(2) PbS, $PbSO_4$, $PbCrO_4$

(3) PbS, $PbSO_4$, $Pb(CH_3COO)_2$

(4) $PbCl_2$, $Pb(SO_4)_2$, $PbCrO_4$

Ans. (2)

Sol.

$$\underset{\boxed{A}}{\boxed{PbS}} \xrightarrow{HNO_3} Pb(NO_3)_2 \xrightarrow{H_2SO_4} \underset{\boxed{B}}{\boxed{PbSO_4}} \xrightarrow[\text{(3) } K_2CrO_4]{\text{(1) Ammonium acetate (2) Acetic acid}} \underset{\boxed{C}}{\boxed{PbCrO_4}}$$

Yellow ppt

57. Given below are two statements:

Statement (I): The boiling points of alcohols and phenols increase with increase in the number of C-atoms.

Statement (II): The boiling points of alcohols and phenols are higher in comparison to other class of compounds such as ethers, haloalkanes.

In the light of the above statements, choose the **correct** answer from the options given below:

(1) Both **Statement I** and **Statement II** are false

(2) **Statement I** is false but **Statement II** is true

(3) **Statement I** is true but **Statement II** is false

(4) Both **Statement I** and **Statement II** are true

Ans. (4)

Sol. B.P. $\propto$ M.W.

B.P. $\propto$ Inter molecular hydrogen bonding

Alcohol & Phenol have intermolecular H-bonding

58. When a non-volatile solute is added to the solvent, the vapour pressure of the solvent decreases by 10 mm of Hg. The mole fraction of the solute in the solution is 0.2. What would be the mole fraction of the solvent if decrease in vapour pressure is 20 mm of Hg ?

(1) 0.6 (2) 0.4

(3) 0.2 (4) 0.8

Ans. (1)

Sol. $\because P^\circ - P \propto X_{solute}$

and $\because 10 \propto 0.2$

$\therefore 20 \propto 0.4$

$\therefore X_{solvent} = 1 - X_{solute}$

$= 1 - 0.4$

$= 0.6$

59. Given below are two statements:

Statement (I) : For a given shell, the total number of allowed orbitals is given by n^2.

Statement (II) : For any subshell, the spatial orientation of the orbitals is given by $-l$ to $+l$ values including zero.

In the light of the above statements, choose the **correct** answer from the options given below:

(1) **Statement I** is true but **Statement II** is false

(2) **Statement I** is false but **Statement II** is true

(3) Both **Statement I** and **Statement II** are true

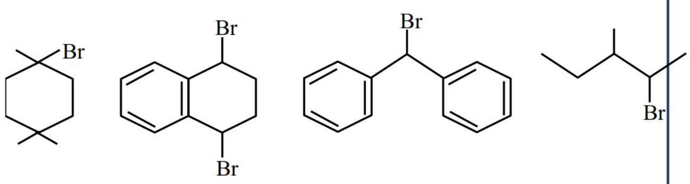
(4) Both **Statement I** and **Statement II** are false

Ans. (3)

Sol. For a shell total number of orbitals = n^2

Magnetic quantum number have values ($-\ell$ to $+\ell$) including 0.

60. The ascending order of relative rate of solvolysis of following compounds is



(A) (B) (C) (D)

(1) (D) < (A) < (B) < (C)

(2) (C) < (B) < (A) < (D)

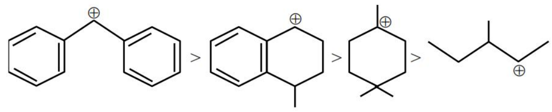
(3) (D) < (B) < (A) < (C)

(4) (C) < (D) < (B) < (A)

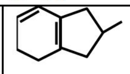
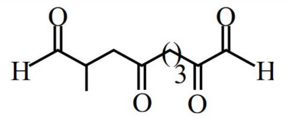
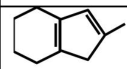
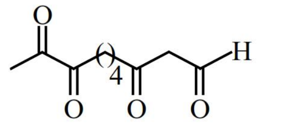
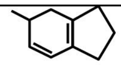
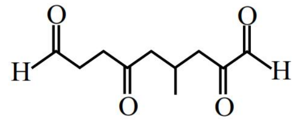
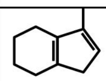
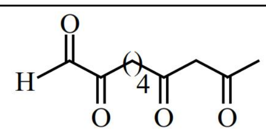
Ans. (1)

Sol. Solvolysis or $S_N1 \propto$ stability of carboccation

Stability order



61. Match **List - I** with **List - II**.

List - I (Isomers of $C_{10}H_{14}$)		**List - II (Ozonolysis product)**	
(A)		(I)	
(B)		(II)	
(C)		(III)	
(D)		(IV)	

Choose the **correct** answer from the options given below :

(1) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

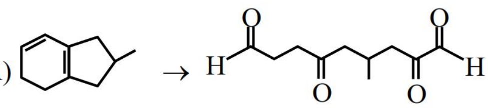
(2) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

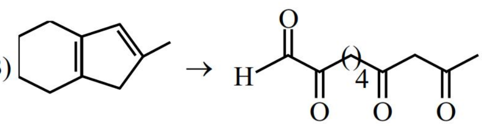
(3) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

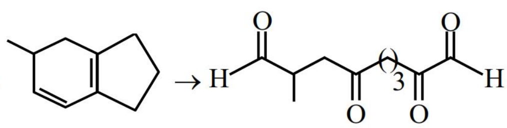
(4) (A)-(I), (B)-(IV), (C)-(III), (D)-(II)

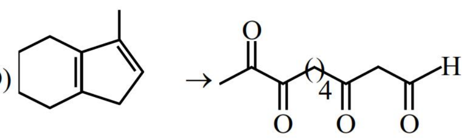
Ans. (2)

Sol. Ozonolysis product

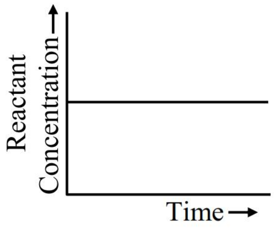
(A) 

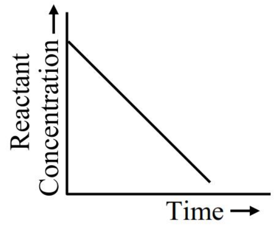
(B) 

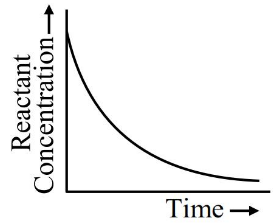
(C) 

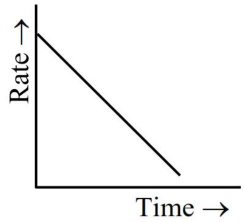
(D) 

62. Which of the following graphs most appropriately represents a zero order reaction ?

(1)


(2)


(3)


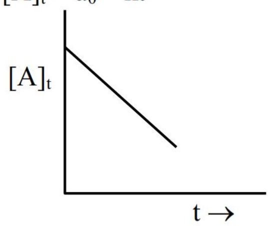
(4)


Ans. (2)

Sol. For zero order reaction : $A \rightarrow P$

Rate = k

$[A]_t = a_0 - kt$



63. Match **List - I** with **List - II**.

List - I		**List - II**	
(A)	Bronze	(I)	Cu, Ni
(B)	Brass	(II)	Fe, Cr, Ni, C
(C)	UK silver coin	(III)	Cu, Zn
(D)	Stainless Steel	(IV)	Cu, Sn

Choose the **correct** answer from the options given below :

(1) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)
(2) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)
(3) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)
(4) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

Ans. (2)

Sol. Bronze $\rightarrow$ Cu, Sn

Brass $\rightarrow$ Cu, Zn

UK silver coin $\rightarrow$ Cu, Ni

Stainless steel $\rightarrow$ Fe, Cr, Ni, C

64. Identify the coordination complexes in which the central metal ion has d^4 configuration.

(A) $[FeO_4]^{2-}$
(B) $[Mn(CN)_6]^{3-}$
(C) $[Fe(CN)_6]^{3-}$
(D) $Cr_2(O-\overset{\overset{O}{||}}{C}-Me)_4(H_2O)_2$
(E) $[NiF_6]^{2-}$

Choose the **correct** answer from the options given below :

(1) (C) and (E) only
(2) (B), (C) and (D) only
(3) (B) and (D) only
(4) (A), (B) and (E) only

Ans. (3)

Sol. $Fe^{+6} = [Ar]3d^2$

$Mn^{+3} = [Ar]3d^4$

$Fe^{+3} = [Ar]3d^5$

$Cr^{+2} = [Ar]3d^4$

$Ni^{+4} = [Ar]3d^6$

65. Given below are the atomic numbers of some group 14 elements. The atomic number of the element with lowest melting point is :

(1) 14
(2) 6
(3) 82
(4) 50

Ans. (4)

Sol. Order of M.P. of group 14 : C > Si > Ge > Pb > Sn

element	M.P. (°C)
Z = 6 = C	3730
Z = 14 = Si	1410
Z = 32 = Ge	937
$\boxed{Z = 50}$ = Sn	232
Z = 82 = Pb	327

66. pH of water is 7 at 25°C. If water is heated to 80°C, it's pH will :

(1) Decrease
(2) Remains the same
(3) H^+ concentration increases, OH^- concentration decreases
(4) Increase

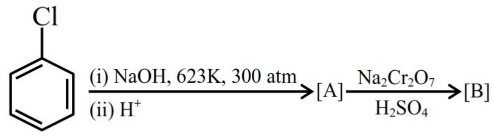
Ans. (1)

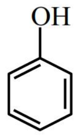
Sol. With increase in temperature, K_w of water increases

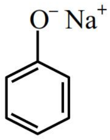
So, degree of dissociation of water increase

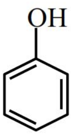
$\therefore$ pH as well as pOH of water decrease.

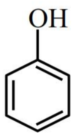
67. Identify the products [A] and [B], respectively in the following reaction :

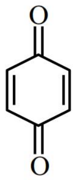


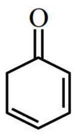
(1) [A] 

, [B] 

(2) [A] 
, [B] 

(3) [A] 

, [B] 

(4) [A] 
, [B] 

Ans. **(3)**

Sol. A is phenol and B is para benzoquinone.

68. Consider a binary solution of two volatile liquid components 1 and 2 x_1 and y_1 are the mole fractions of component 1 in liquid and vapour phase, respectively. The slope and intercept of the linear plot of $\frac{1}{x_1}$ vs $\frac{1}{y_1}$ are given respectively as :

(1) $\frac{P_1^0}{P_2^0}, \frac{P_2^0 - P_1^0}{P_2^0}$ (2) $\frac{P_2^0}{P_1^0}, \frac{P_1^0 - P_2^0}{P_2^0}$

(3) $\frac{P_1^0}{P_2^0}, \frac{P_1^0 - P_2^0}{P_2^0}$ (4) $\frac{P_2^0}{P_1^0}, \frac{P_2^0 - P_1^0}{P_2^0}$

Ans. **(1)**

Sol. $\because$ For liquid solution of two liquids '1' and '2'

$P_1 = P_T y_1 = P_1^o x_1$

$$\therefore \frac{P_T}{x_1} = \frac{P_1^o}{y_1}$$

$$\therefore \frac{P_2^o + x_1(P_1^o - P_2^o)}{x_1} = \frac{P_1^o}{y_1}$$

$$\therefore \frac{P_2^o}{x_1} + (P_1^o - P_2^o) = \frac{P_1^o}{y_1}$$

$$\therefore \frac{1}{x_1} = \left(\frac{P_1^o}{P_2^o}\right)\left(\frac{1}{y_1}\right) + \left(\frac{P_2^o - P_1^o}{P_2^o}\right)$$

$$\therefore \text{Slope} = \left(\frac{P_1^o}{P_2^o}\right)$$

$$\therefore \text{Intercept} = \left(\frac{P_2^o - P_1^o}{P_2^o}\right)$$

69. Given below are two statements about X-ray spectra of elements :

Statement (I) : A plot of $\sqrt{\nu}$ (ν = frequency of X-rays emitted) vs atomic mass is a straight line.

Statement (II) : A plot of ν(ν = frequency of X-rays emitted) vs atomic number is a straight line.

In the light of the above statements choose the **correct** answer from the options given below :

(1) **Statement I** is true but **Statement II** is false

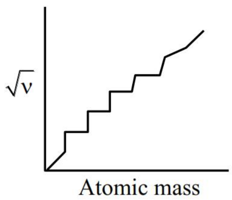
(2) Both **Statement I** and **Statement II** are true

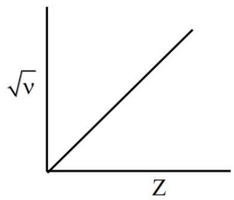
(3) Both **Statement I** and **Statement II** are false

(4) **Statement I** is false but **Statement II** is true

Ans. **(3)**

Sol.





70. Consider the following reactions

$$K_2Cr_2O_7 \xrightarrow[-H_2O]{KOH} [A] \xrightarrow[-H_2O]{H_2SO_4} [B] + K_2SO_4$$

The products [A] and [B], respectively are :

(1) $K_2Cr(OH)_6$ and Cr_2O_3

(2) K_2CrO_4 and Cr_2O_3

(3) K_2CrO_4 and $K_2Cr_2O_7$

(4) K_2CrO_4 and CrO

Ans. **(3)**

Sol. $K_2Cr_2O_7 \xrightarrow[-H_2O]{KOH} \underset{[A]}{K_2CrO_4} \xrightarrow[-H_2O]{H_2SO_4} \underset{[B]}{K_2Cr_2O_7} + K_2SO_4$

SECTION-B

71. 0.01 mole of an organic compound (X) containing 10% hydrogen, on complete combustion produced 0.9 g H_2O. Molar mass of (X) is ______ g mol^{-1}.

Ans. (100)

Sol. Organic compound $\xrightarrow{\text{combustion}} \underset{0.9\,gm}{H_2O}$

$\therefore$ mole of $H_2O = \dfrac{0.9}{18} = 0.05$ mole

$\therefore$ mole of H in $H_2O = 0.05 \times 2 = 0.1$ mole
= mole of H in 0.01 mole Organic compound

$\therefore$ wt of H atom in 0.01 mole compound = 0.1×1 = 0.1 gm

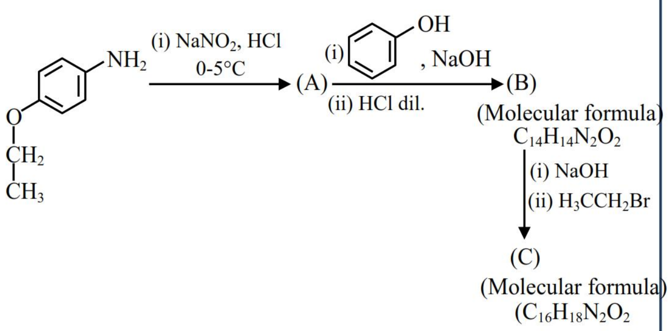
$\therefore$ wt of H atom in one mole compound $= \dfrac{0.1}{0.01} = 10\,gm$

$\because$ wt. % of H = $\dfrac{\text{wt. of H in one mole compound}}{\text{Molar mass of compound}} \times 1$

$10 = \dfrac{10}{M} \times 100$

$\therefore$ $\boxed{M = 100}$

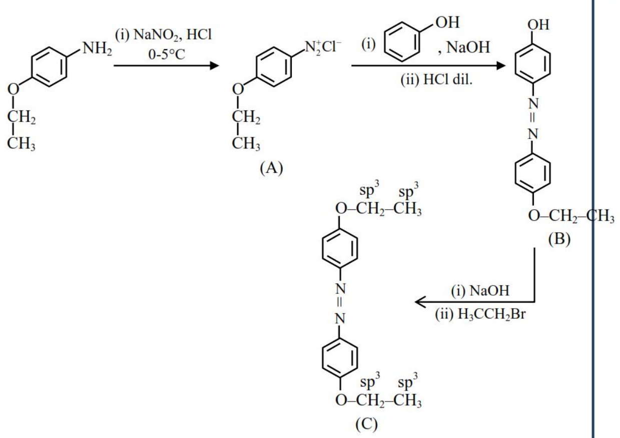
72. Consider the following sequence of reactions.



Total number of sp^3 hybridised carbon atoms in the major product C formed is ________.

Ans. (4)

Sol.



73. When 81.0 g of aluminium is allowed to react with 128.0 g of oxygen gas, the mass of aluminium oxide produced in grams is ____. (Nearest integer)

Given :

Molar mass of Al is 27.0 g mol^{-1}

Molar mass of O is 16.0 g mol^{-1}

Ans. (153)

Sol. $4Al + 3O_2 \longrightarrow 2Al_2O_3$

$\dfrac{81}{27} = 3\,mole \qquad \dfrac{128}{32} = 4\,mole$

Limiting reagent

$\therefore$ mole of Al_2O_3 formed = $\dfrac{1}{2} \times 3$ mole

$\therefore$ wt. of Al_2O_3 formed = $\dfrac{3}{2} \times 102$

= 153 gm

74. The bond dissociation enthalpy of X_2 ΔH^o_{bond} calculated from the given data is _______ kJ mol^{-1}. (Nearest integer)

$M^+X^-(s) \rightarrow M^+(g) + X^-(g)$ $\Delta H°_{lattice} = 800$ kJ mol^{-1}

$M(s) \rightarrow M(g)$ $\Delta H°_{sub} = 100$ kJ mol^{-1}

$M(g) \rightarrow M^+(g)^- + e^-(g)$ $\Delta H°_i = 500$ kJ mol^{-1}

$X(g) + e^-(g) \rightarrow X^-(g)$ $\Delta H°_{eg} = -300$ kJ mol^{-1}

$M(s) + \dfrac{1}{2}X_2(g) \rightarrow M^+X^-(s)$ $\Delta H°_f = -400$ kJ mol^{-1}

[Given : M^+X^- is a pure ionic compound and X forms a diatomic molecule X_2 is gaseous state]

Ans. (200)

Sol.

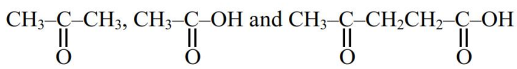
$$M_{(s)} + \frac{1}{2}X_{2(g)} \longrightarrow M^{\oplus}X^{\ominus}_{(s)}$$
$$\downarrow \qquad\qquad \downarrow$$
$$M_{(g)} \qquad X_{(g)}$$
$$\downarrow \qquad\qquad \downarrow$$
$$M^+_{(g)} + X^-_{(g)} \longrightarrow M^{\oplus}X^{\ominus}_{(s)}$$

$\therefore \Delta H_f(MX) = \Delta H_{sub}(M) + I.E.(M) + \dfrac{1}{2}[B.E.(X-X)]$
$+ EG(X) + L.E.(MX)$

$-400 = (100) + (500) + \dfrac{1}{2}(B.E.) + (-300) + (-800)$

$\therefore$ B.E. = 200 kJ $mole^{-1}$

75. A compound 'X' absorbs 2 moles of hydrogen and 'X' upon oxidation with $KMnO_4 \mid H^+$ gives



The total number of σ bonds present in the compound 'X' is ________.

Ans. **(27)**

Sol.